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ON THE MAGNETIC BEHAVIOUR OF SUPERCONDUCTING LEAD DISCS

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ABSTRACT

An apparatus for measuring static magnetic susceptibilities has been used for studying the magnetic behaviour of superconducting samples in gradients of low ($\leq$ 10 mT) magnetic induction fields. Forces on 3 mm diameter thin lead discs at temperatures down to 2.5 K have been measured with a sensitive (1 nN) vacuum microbalance. The interpretation of the data reveals that the three phenomena i) prohibition of flux penetration, ii) Meissner-Ochsenfeld flux expulsion and iii) flux trapping, occurring in lead are similar to those observed in vanadium (ref. 1). In the lead samples, however, saturation effects are revealed.

INTRODUCTION

The striking part of the apparatus for measuring static magnetic susceptibilities on small solid samples is a vacuum microbalance, with a sensitivity about 1 nanonewton (ref. 2). Forces exerted on thin superconducting discs by small non-uniform magnetic fields, have been measured with the balance. Reference 3 reports on experiments which had been carried out on 3 mm diameter vanadium discs at 3 K in magnetic induction fields lower than 8 millitesla. The analysis of the data reveals that i) the reproducibility of the force measurement at low temperature can be better than 0.04 %; and ii) the magnetic moment of the vanadium consists of two parts: an induced one and a permanent one. The former part is due to the measuring field while the latter part was trapped during cooling of the sample. The trapped permanent moment, in the vanadium discs, remained constant during the experiment.

The aim of the present paper is to report on similar experiments which have been carried out on lead discs. It will be shown that the magnetic behaviour of the lead samples is analogous to that of vanadium but with greater complexity.

EXPERIMENTS AND RESULTS

The susceptibility measurements for the present study have been carried out on lead discs which had been punched out of thin foils of very pure metal (ref. 4). Sample data are given in Table 1.

The average sample thicknesses t are calculated from the sample mass, m,

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TABLE 1
Lead discs data

sample nr	m(ass) (mg)	d(iameter) (mm)	t(hickness) (μm)
666	8.90	3.02	110.
667	0.56	2.98	7.
668	8.72	3.01	108.
671	2.01	3.02	25.

and the density of bulk lead, $\rho = 11.3 \times 10^3$ kg m^{-3}, by $t = m/\rho S$ with $S = \pi d^2/4$.

The lead disc lies on the balance pan hanging in the dewar tail. On the tail a small ironless coil is mounted. Its current generates an applied field H, which is perpendicular to the disc surface. The coil/pan situation is drawn in figure 1 in reference 3. All experiments for the present study have been carried out with the lead discs at the same position on the coil axis, P = 11.0 mm (ref. 3). The applied field at the sample therefore is proportional to the coil current I: $H = H_1.I = 8.00 \times 10^{+3} \times I$ (Am^{-1}). The gradient of the applied induction field at the sample also is proportional to I: $G = \mu_0(dH/dz) = G_1.I = -0.464.I$ (Tm^{-1}), with μ_0 the free space permeability. Each experiment started by cooling the sample from above 7.5 K down to the measuring temperature T_m while the coil current was kept constant at I_c. When the Meissner-Ochsenfeld effect was considered, the force change $F_c = F_I - F_o$ was obtained for a reduction of the coil current from I_c to zero, at the measuring temperature T_m. Subsequently force measurements are carried out in each experiment for the coil current I_m in a chopping mode, alternating the current direction. The procedure allows to analyse the lead data in the same way the vanadium data were treated. In vanadium it was found that reversal of the coil current direction allowed for distinguishing between the induced moment and the permanent one.

The measured sample force is expressed as:

$$F = m\,(\chi H + P)G \tag{1}$$

where χ is the susceptibility and P the permanent dipole moment, both per unit of mass. This equation can be written as:

$$F = F_A + F_B = AI_m^2 + BI_m \tag{2}$$

$$\text{with } A = m\chi H_1 G_1 \tag{3}$$

$$\text{and } B = mPG_1 \tag{4}$$

where A and B are calculated using the balance output voltage V and the balance constant $C = 5.14 \times 10^{5}\ VN^{-1}$, by:

$$A = (V(+)+V(-))/(2I_m^2C) \tag{5}$$
$$B = (V(+)-V(-))/(2|I_m|C) \tag{6}$$

The signs (+) and (-) refer to the polarity of the coil current I_m. The analysis holds for χ and P remaining constant during the measurements.

In a first group of experiments the lead discs were allowed to cool down to $T_m = 4.37\ (\pm\ 0.07)$K at $I_c = 0$ A. For each sample the data have been gathered during series of cycle trains. One cycle train consists of a few consecutive current reversing cycles during which the amplitude of the measuring current $|I_m|$ has been kept constant. A measuring series is constituted of a number of cycle trains for which the $|I_m|$ values increase, from 0.100 A to the maximum value, in steps of 0.100 A. The values of $\bar{A}$ and $\bar{B}$, obtained for each cycle train, are plotted, respectively in Fig. 1 and in Fig. 2, versus the measuring coil-current $|I_m|$, by the crosses + for sample number 666, by $\dagger$ for 667, by $\times$ for 668, by $\triangle$ for 671 first series and by $\odot$ for 671 second series. For the samples 666, 667 and 668 only the data of their first series are discussed. For sample 671 a complete second measuring series has been taken right after the last measurement of the first series for which $I_m = -\ 0.900$ A. The $\bar{A}$ values of sample 666 for $|I_m| \geq 0.200$ A, fit the equation:

$\bar{A} \cong 3.759\ (1-0.01116.|I_m|) \times 10^{-5}$ to
$\sigma \leq 0.002 \times 10^{-5}$ so that $\zeta = (\sigma/\bar{A}) \leq 0.05\ \%$
The data of the sample 668 yield
$\bar{A} \cong 3.784\ (1-0.01586.|I_m|) \times 10^{-5}$
$\sigma \leq 0.003 \times 10^{-5}$ and $\zeta \leq 0.07\ \%$

In a second group of experiments, the lead disc 671 has been cooled down to $T_m = 2.62\ (\pm\ 0.01)$K in the field generated by the coil current $I_c = 0.400$ A. In the first series of cycle trains the coil current $|I_m|$ was pushed up from 0.100 A to 0.900 A in steps of 0.100 A. During the last cycle train the last measurement has been carried out at $I_m = -\ 0.900$ A. Then the second series of cycle trains has been started again at $|I_m| = 0.100$ A. The resulting $\bar{A}$ and $\bar{B}$ values are plotted by the crosses + for the first series and by $\dagger$ for the second series in respectively Fig. 3 and Fig. 4 versus $|I_m|$. For comparison the data have been added which had been taken on the same sample at $T_m = 4.32$ $(\pm\ 0.02)$K in the first group of experiments. As may be seen from Fig. 3 and 4 a lower temperature corresponds with values of $\bar{A}$ which are larger at higher $|I_m|$ and with off zero values of $\bar{B}$ that remain higher up to higher $|I_m|$.

Neither $\bar{A}$ nor $\bar{B}$ remained constant in the lead disc 671, as distinct from the vanadium results described in reference 3. The question arises whether it is an acceptable approximation or not to consider A and B to be constant also in the case of lead. The answer is given by considering the second series of cycle trains which have been measured on the same sample 671 right after the measurement of the first series in the second group of experiments. In the first train I_m = - 0.100 A, + 0.100 A, - ..., + 0.100 A. The analysis of the data yields $\bar{A} = 3.33\times10^{-5}$ while B evolves from 0.960×10^{-5} to 0.951×10^{-5}. In the second train I_m = - 0.200 A (q=1), + 0.200 A (q=2), - ..., + 0.200 A (q=24). In Fig. 5 and Fig. 6 respectively the calculated values for A and B are plotted versus the sequential number q of the measurements. The analysis yields $\bar{A} = 3.34\times10^{-5}$ NA^{-2} with B evolving from 0.891×10^{-5} to 0.810×10^{-5} NA^{-1}. The temperature remained nearly constant during the cycle train; T_m = 2.65 (± 0.01)K. The figures clearly show that A and B at the first measurement are different from the other values in the cycle train. After a few measurements, however, the values of both, A and B, remained constant.

In a third group of experiments the lead disc, sample number 668, has been cooled to T_m = 2.8 K in the field of the coil current I_c, for the series in which I_c = 0.200 A, 0.400 A, 0.600 A, 0.800 A, 0.900 A, 0.700 A, 0.500 A, 0.300 A and 0.100 A in the order given. After each cooling first the force change F_c, for the coil current reduction from I_c to zero, has been measured. Then the $\bar{A}$ and $\bar{B}$ constants have been determined at the same temperature with I_m = ± 0.060 A. From the data the susceptibilities were calculated by:

$$\chi_A = A/(mH_1G_1) \quad (7)$$
$$\chi_B = B/(mH_1G_1I_c) \quad (8)$$
$$\text{and } \Sigma = \chi_M - \chi_B \quad (9)$$
$$\text{with } \chi_M = F_c/(mH_1G_1I_c^2) \quad (10)$$

The values of - χ_A, χ_B and - Σ are plotted versus I_c in Fig. 7. In contrast with the earlier observation that $\Sigma = \chi_A$ in the vanadium case, in the lead sample $|\Sigma| < |\chi_A|$.

In a fourth group of experiments, the lead sample 666 has been cooled down to T_m = 2.8 K in the field of the coil current I_c = 0.400 A. The force F, as measured for I_m = ± 0.100 A at T_m increasing from 2.8 K to above the critical temperature T_{co} = 7.2 K, is plotted in Fig. 8 versus T_m. The analysis yields a permanent moment P the magnitude of which decreases with increasing temperature. In the temperature range where B differs from zero, A is constant. In the region near the transition temperature, where B vanishes, A decreases with increasing temperature. A becomes zero at the transition temperature T_{cI} in the field generated by the measuring coil current I_m.

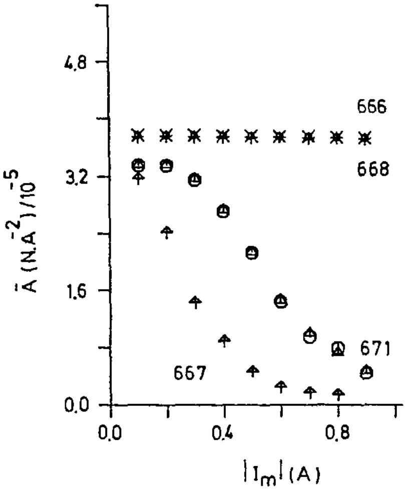

Fig. 1. Induced moment parameter A, measured at 4.3 K, vs coil current.

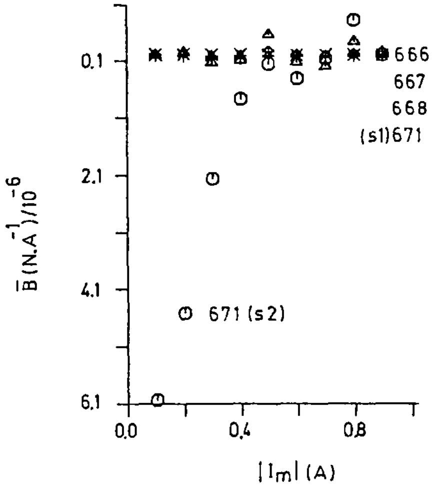

Fig. 2. Permanent moment parameter B, measured at 4.3 K, vs coil current.

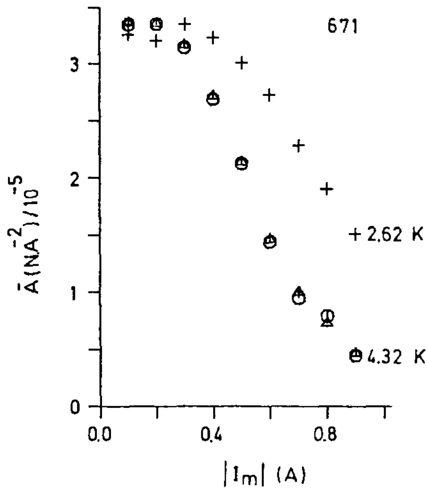

Fig. 3. Induced moment parameter A, measured at 2.6 K (+,↑) and at 4.3 K, vs coil current intensity.

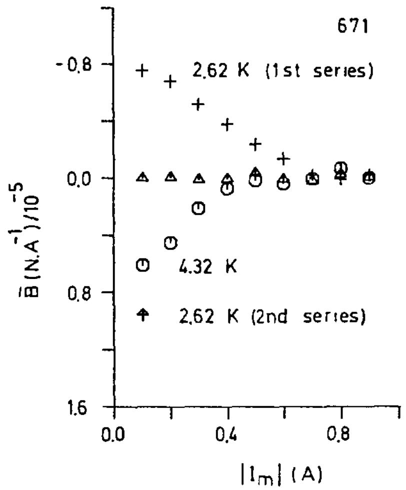

Fig. 4. Permanent moment parameter B, measured at 2.6 K (+,↑) and at 4.3 K, vs coil current intensity.

On the same sample 666 the forces F have been measured for different $|I_m|$ = 0.100 A, 0.300 A, 0.400 A, 0.500 A, 0.600 A and 0.700 A. A linear fit of the forces F, in the range 20 nN < F < 80 nN, related to one value of $|I_m|$, versus temperature yields the value $T_{cI}(I_m)$ by extrapolation to F = 0. On their turn, these T_{cI} values fit a linear equation which, on putting $|I_m|$ = 0, yields T_{co} = 7.19 K. The value is in good agreement with the one published in reference 4. In the same way the critical temperature of different pure lead discs have been measured over a period of 10 years, resulting in T_{co} = 7.20 (± 0.02)K.

DISCUSSION

The first group of experiments on the lead discs reveal that the measurement of the induced moment parameter A is reproducible. Indeed, for sample number 671 the values measured in a second series of cycle trains are the same as those measured in the first series of cycle trains at the related coil current intensities. It is shown in Fig. 1 by the superposition of the ⊙ signs on the △'s. By the superposition of the ☒'s over the + crosses, which signs are referring to the samples 668 and 666 respectively, moreover Fig. 1 reveals that for two different samples of the same thickness the A values, related to each coil current intensity, are also the same. The reproducibility is an indication for the reliability of the measurements. The large differences in Fig. 1 of the A values at a certain $|I_m|$, are related to the thickness of the sample discs as all discs have about the same diameter. The thinner the sample the smaller the A value is.

Up till now, only experimental facts have been reported. Force measurements carried out on superconducting lead plates which were oriented perpendicular to the field have been described. The analyses resulted in values for the sample's macroscopic magnetic moment or susceptibility. In what follows it will be attempted to correlate the macroscopic effects in the light of the rich variety of phenomena superconductors display in an applied field (e.g. ref. 5). Because the forces on the sample are due to the interaction of the superconducting currents in the samples with the coil current, one can argue that the effect is due to current limitations in the samples related to critical current densities of the material, here of the pure lead. In this model the observation that, in the thicker discs for the range of the $|I_m|$ values used, the value of A is nearly constant indicates that the superconducting currents in the material might have suffered only a slight decrease. In the limiting case of high critical current densities and low superconducting current intensities in the sample, the prohibition of flux penetration resulting from the zero resistance of the material implies that all sample currents are flowing near the surface. In this approximation the

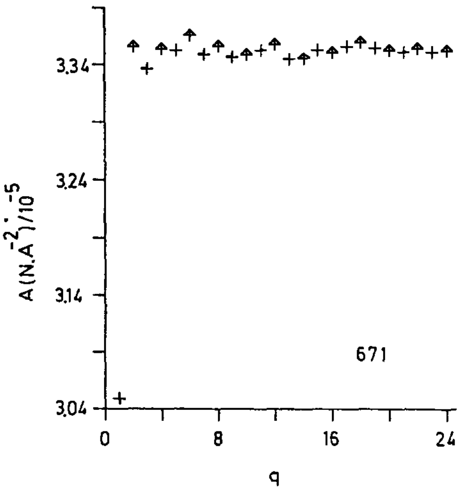

Fig. 5. Induced moment parameter A vs measuring number.

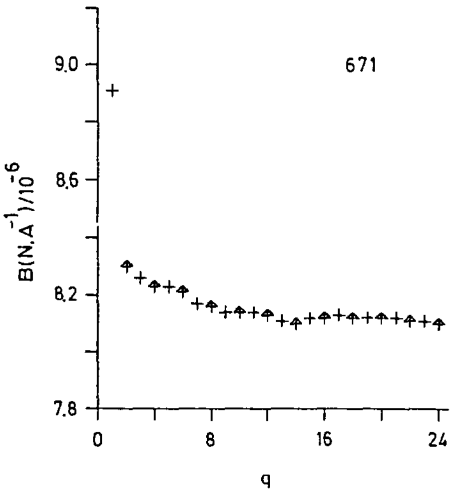

Fig. 6. Permanent moment parameter B vs measuring number.

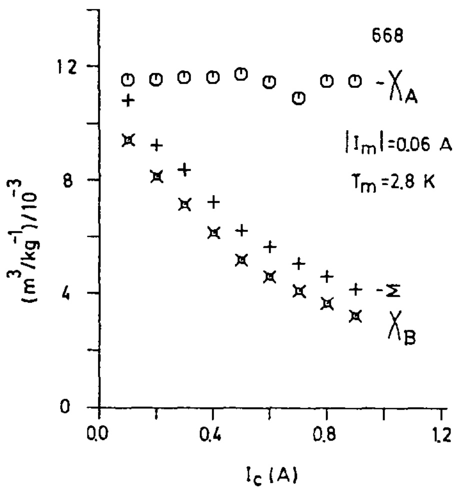

Fig. 7. The susceptibility of sample 688.

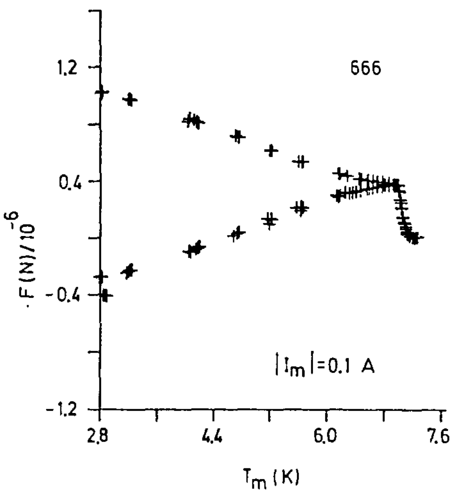

Fig. 8. Vertical force, measured for increasing temperatures from 2.8 K, vs temperature.

value of the induced moment parameter A relates only to the geometry of the sample, being independent of the sample's material. Indeed, the value of A for a 3.0 mm diameter vanadium disc (ref. 3) compares well to the maximum value of A of the lead sample 668; i.e. 4.07×10^{-5} NA^{-2} and 3.77×10^{-5} NA^{-2}, respectively. The difference of 8 percent might be related to the fact that the vanadium disc is by 87 µm thicker than the considered lead disc. A similar thickness dependence is observed for the lead samples considered in Fig. 1. The maximum value of A for the 25 µm thick sample is by 4×10^{-6} NA^{-2} lower than the related A value of sample 668. The thickness effect can be estimated from the oversimplified model in which the force on the sample is considered to be due to two parts of the superconducting currents: those flowing on the horizontal top and bottom surface and those flowing on the vertical cylinder surface. It is further assumed that, to a good approximation, the top and bottom terms can be considered to be the same for all samples of the same diameter. The thickness effect should then come from the vertical cylinder surface. The difference in thickness between the samples 668 and 671 is Δt = 83 µm. For this the calculated difference in A is $\Delta A = H_1.\Delta t.S.G_1$ (= 2.2×10^{-6} NA^{-2}), which is of the right order in magnitude of the measured difference. The precision on the measured differences is not very high because of the difficulty in positioning the disc's flat surface perpendicular to the field direction. The projection of the sample's surface on the plane perpendicular to the field easily may be smaller than the sample's surface by about 2 percent.

The B values, calculated from the data taken on the lead discs in the first group of experiments and given in Fig. 2, do not reveal persistent moments in the first series of measuring cycle trains. In the second series, however, permanent dipoles are observed at the lower $|I_m|$ measurements. The moment appeared in the sample for which the last measurement in the first series resulted in an $\bar{A}$ value which is smaller than $\bar{A}$ maximum. The latter effect, $(\bar{A}/\bar{A}_{max}) < 1$, has been interpreted above as due to the limitations on the shielding currents by critical values with the consequence that flux penetration into the sample occurs.

The persistent moment in the lead disc is not so permanent as the one measured in the vanadium (ref. 3). Indeed, in the lead disc 671 it decreases from cycle train to cycle train, showing that measuring of the moment can influence the moment itself.

In Fig. 3 the values of $\bar{A}$, as measured on the lead disc 671 for two different temperatures, are practically the same in the lower $|I_m|$ region. The "geometry" seems to dominate the situation implying that the inner part of the sample is well shielded for the applied magnetic induction field. At the higher values of $|I_m|$ the smaller values of $\bar{A}$ indicate that for both temperatures the shielding currents are limited by critical values. The limitations seem to be less severe at the lower measuring temperature than at the higher one. The result is in good agreement with the expected temperature dependence of critical currents. The critical current behaviour suggests that the persistence of the trapped dipole moment in the lead disc should be higher at the lower temperature. Indeed, as shown in Fig. 4, the dipole moment trapped during cooling of the sample 671 in the field of I_c = 0.400 A lasted to a higher value of the measuring $|I_m|$ at 2.6 K than did the trapped flux at 4.3 K. The flux trapped at the last measurement of the first cycle train series is larger at 2.6 K (+) than at 4.3 K (⊙). Moreover the experiments reveal that the sign of the persistent dipole is related to the direction of the field that penetrated in the sample.

The decrease of the persistent moment with increasing value of $|I_m|$ raised the question whether or not B but also A remains constant in each cycle train in the case of lead. The Fig. 5 and Fig. 6, which are related to the case of low measuring coil current and rather high trapped dipole moment, show that the very first calculated values of both A and B differ from the following ones in the cycle train. The effect suggests that there has been a mutual perturbation. The second and following measurements yield A values that very closely approximate the purely geometrical one. The influence of A on B, however, seems to last for more cycles, as shown in Fig. 6. For both entities A and B it can be stated that after a few measurements their values remain constant during the cycle train.

Measuring with an increased coil current intensity seems to be able to wipe out part of the pre-existing persistent moment. The extrapolation of this statement to higher $|I_m|$ values, suggests that in the higher field case the induced currents may "overwrite" formerly existing superconductive currents. When the induced currents become so high that the addition and subtraction of the persistent currents still result in currents higher than allowed for by the critical values then the limited currents will have the same intensities for both coil current directions. Then the calculated values of B will be very low as shown in Fig. 2 and Fig. 4.

The susceptibilities of the lead disc versus I_c, as given in Fig. 7, yield a pattern different from that of vanadium (ref. 1). The similarity between both patterns is the best in the low field region. In the measurement for which I_c = 0.100 A, $|\Sigma|$ is smaller than $|\chi_A|$ by only 7 percent. For increasing field the fraction $|\Sigma| / |\chi_A|$ becomes smaller to reach the value of 0.36 for I_c = 0.900 A. The fraction $|\chi_M| / |\chi_A|$, for χ_A measured with $|I_m|$ = 0.060 A, decreases also with increasing I_c; from 0.12 at I_c = 0.100 A to 0.08 at I_c = 0.900 A. It indicates that the Meissner-Ochsenfeld part, $|\chi_M|$, of $|\Sigma|$ is less sensitive to I_c than χ_B. The decrease of $|\Sigma|$ with increasing field therefore mainly is attributed to the decrease of χ_B, i.e. to the limitation of the persistent currents in the sample.

In Fig. 8 the forces measured on sample 666 reveal that the permanent moment decreases while the temperature increases during the measurements. Here again the reduction of the moment is in agreement with the reduction of the critical current values at increasing temperature.

CONCLUDING REMARKS

Force measurements, which have been carried out on superconducting lead discs that were oriented perpendicular to the applied field, have been reported here. The analysis of the data resulted in values for the macroscopic magnetic moments and susceptibilities of the samples. On cooling the discs in a magnetic induction field from above to below T_c a Meissner-Ochsenfeld effect is observed. The interpretation of the data yields partial (≅ 10 %) flux expulsion. That the flux is not completely expelled from the sample is understood on the basis of the flatness of the sample. The flux part, which had not been expelled by the Meissner-Ochsenfeld effect, has been trapped in the cooled superconducting material. The trapped flux yields a moment that has been measured. It can be overwritten by the currents induced at their measurement. The behaviour of the induced moment parameter A, at the lower fields in the thicker discs, suggests that its value related only to the geometry of the sample. This model is in agreement with the zero resistance of the material which prohibits new flux to penetrate the sample. For higher measuring fields flux penetration has been measured to occur. It is attributed to a reduced sample shielding caused by current limitations. The magnetic behaviour of the superconducting lead discs, in the low field thicker sample regime , is very similar to that of the vanadium samples (ref. 1). In the higher field and thinner sample regimen it is more complex. The superconducting lead discs show saturation effects and reveal a history dependence which both can be qualitatively understood on the basis of superconducting current limitations by critical values.

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